

NAG Toolbox for MATLAB

f04ah

1 Purpose

f04ah calculates the accurate solution of a set of real linear equations with multiple right-hand sides, $AX = B$, with iterative refinement, where A has been factorized by f03af.

2 Syntax

```
[x, bb, k, ifail] = f04ah(a, aa, p, b, eps, 'n', n, 'ir', ir)
```

3 Description

To solve a set of real linear equations $AX = B$, f04ah must be preceded by a call to f03af which computes an LU factorization of A with partial pivoting, $PA = LU$, where P is a permutation matrix, L is lower triangular and U is unit upper triangular. An approximation to X is found by forward and backward substitution. The residual matrix $R = B - AX$ is then calculated using *additional precision*, and a correction D to X is found by solving $LUD = PR$. X is replaced by $X + D$, and this iterative refinement of the solution is repeated until full machine accuracy has been obtained.

4 References

Wilkinson J H and Reinsch C 1971 *Handbook for Automatic Computation II, Linear Algebra* Springer-Verlag

5 Parameters

5.1 Compulsory Input Parameters

- 1: **a(lda,n) – double array**
lda, the first dimension of the array, must be at least **n**.
 The n by n matrix A .
- 2: **aa(ldaa,n) – double array**
ldaa, the first dimension of the array, must be at least **n**.
 Details of the LU factorization, as returned by f03af.
- 3: **p(n) – double array**
 Details of the row interchanges as returned by f03af.
- 4: **b(ldb,ir) – double array**
ldb, the first dimension of the array, must be at least **n**.
 The n by r right-hand side matrix B .
- 5: **eps – double scalar**
 Must be set to the value of the *machine precision*.

5.2 Optional Input Parameters

1: **n** – **int32 scalar**

Default: The dimension of the arrays **a**, **aa**, **p**. (An error is raised if these dimensions are not equal.)
n, the order of the matrix *A*.

Constraint: $n \geq 0$.

2: **ir** – **int32 scalar**

Default: The dimension of the arrays **b**, **x**, **bb**. (An error is raised if these dimensions are not equal.)

r, the number of right-hand sides.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldaa, ldb, ldx, ldbb

5.4 Output Parameters

1: **x(ldx,ir)** – **double array**

The *n* by *r* solution matrix *X*.

2: **bb(ldbb,ir)** – **double array**

The final *n* by *r* residual matrix $R = B - AX$.

3: **k** – **int32 scalar**

The number of iterations needed in the refinement process.

4: **ifail** – **int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

The matrix *A* is too ill-conditioned to produce a correctly rounded solution.

7 Accuracy

The computed solutions should be correct to full machine accuracy. For a detailed error analysis see page 106 of Wilkinson and Reinsch 1971.

8 Further Comments

The time taken by f04ah is approximately proportional to n^2r .

9 Example

```
a = [33, 16, 72;  
      -24, -10, -57;  
      -8, -4, -17];
```

```
aa = [-8, 0.5, 2.125;  
      -24, 2, -3;  
      33, -0.5, 0.375];  
p = [3;  
     2;  
     3];  
b = [-359;  
     281;  
     85];  
eps = 1.111307226797642e-16;  
[x, bb, k, ifail] = f04ah(a, aa, p, b, eps)
```

```
x =  
    1  
   -2  
   -5  
bb =  
    0  
    0  
    0  
k =  
    2  
ifail =  
    0
```